Local Condensing Heat Transfer Coefficients in the Annular Flow Regime

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SCOPE

It has been well documented that the process of condensation of a vapor to a liquid may involve several distinct regimes of flow. The present work concerns itself with one of the simplest conceptually, that of an annular film of condensate on the inside wall of the condenser tube. This liquid film is driven axially along the inner wall of the tube by shear forces exerted on it by the uncondensed portion of the vapor. This process is one of interest in commercial equipment in which high speed vapor is condensed inside cooled tubes.

In this paper some relationships will be derived for the calculation of the thermal resistance of a flowing film of condensate. These relationships are based on a modification of the Martinelli analogy between heat and momentum transfer in turbulent flow. In all analogy methods an independent specification is required of momentum flux transfer. In macroscopic terms this is equivalent to a specification of the axial pressure gradient for the flow. It is not the intention of this paper to present relationships for predicting two-phase pressure drop, but to offer a device for calculating the heat transfer rate given an independent source of information on the two-phase pressure gradient. In the present case an empirical pressure drop relation was used which fit the range of data reported here and that of an independent investigation (*1*).

The limitations on the analytic model are those associated with the requirement of annular flow. Thus condenser orientation is irrelevant, provided sufficient vapor shear forces act on the condensate to maintain annular flow. Excessive vapor shear causes the condensate film to entrain into the vapor core flow. This condition would also violate the prerequisites for the model. Notwithstanding these objections, the range of allowable flow conditions is still substantial (2). It should be emphasized that the method is not limited to the pressure drop range of the data reported here. Provided pressure drop data are available for the system under investigation, the equations developed can be used throughout the annular flow range. These equations are limited to Newtonian liquids where changes in thermophysical properties with temperature are small.

SUMMARY

New data have been taken in annular flow condensation of steam inside an instrumented condenser tube. The following variables were measured: steam flow rate, local quality, local wall temperatures, local heat fluxes, inlet and outlet pressures, and vapor temperatures. In addition, the condenser tube was equipped with a window at both extrema for viewing the flow pattern during condensation. From the raw data local heat transfer coefficients* were calculated.

The pressure drop measurements were broken into their acceleration and frictional components in the conventional manner and a shear velocity was defined from the frictional component

$$v_* = \sqrt{\frac{-D}{4\rho_L}\left(\frac{dp}{dz}\right)_{fr}} \quad (1)$$

The Martinelli analogy of turbulent momentum and heat transfer was then applied to compute the thermal resistance of the condensate film. These results can be expressed in the following nondimensional form

$$\frac{hD}{k_L} = N_{Nu} = \frac{1}{T^+(\delta^+)}\,\frac{\rho_L\, D\, c\, v_*}{k_L} \quad (2)$$

where the nondimensional temperature T^+ is related to the nondimensional film thickness δ^+ by the following equations:

$$T^+(\delta^+) = \delta^+ N_{Pr} \quad \text{for} \quad \delta^+ \leqq 5 \quad (3a)$$

$$T^+(\delta^+) = 5[N_{Pr} + \ln\{1 + N_{Pr}(\delta^+/5 - 1)\}] \quad (3b)$$
$$\text{for} \quad 30 \geqq \delta^+ > 5$$

$$T^+(\delta^+) = 5[N_{Pr} + \ln(1 + 5N_{Pr}) + 0.495 \ln(\delta^+/30)]$$
$$\text{for} \quad \delta^+ > 30 \quad (3c)$$

These are used in conjunction with the nondimensional condensate thickness calculated in terms of the local Reynolds number of the flow from two simple models. These results are

$$\delta^+ = (N_{Re}/2)^{1/2} \quad \text{for} \quad N_{Re} < \sim 1{,}000 \quad (4a)$$

and

$$\delta^+ = 0.0504\, N_{Re}^{7/8} \quad \text{for} \quad N_{Re} > \sim 1{,}000 \quad (4b)$$

where the Reynolds number is defined as

$$N_{Re} = (1 - x)\,\frac{G_T D}{\mu_L} \quad (5)$$

It is shown that this model is an excellent representation of the condensing data. It predicts the substantial change in the local condensing heat transfer coefficient with axial position. In this respect it is more accurate than

* Calculated from the definition $q'' = h\,(T_s - T_w)$.

some existing empirical correlations, including those of Ananiev, Boyko, and Kruzhilin (3, 4) and of Akers, Deans, and Crosser (5). In passing we have noted that the substantial variation in the local condensing heat transfer coefficient, which is characteristic of the condensation of high speed vapor, may require the application of local design methods rather than axially averaged heat transfer coefficients. The empirical equations of references 3, 4, and 5 noted above are known to yield rather accurate estimations of average condensing coefficients in spite of poorer agreement to local coefficients. Their use should therefore be limited to those systems in which the condensing side thermal resistance is a small fraction of the overall resistance.

The shear velocity defined in Equation (1) by analogy to the single-phase turbulent procedure depends on the frictional component of the total pressure gradient. This leads to a satisfactory calculation of heat transfer coefficients in two-phase systems involving momentum changes. Finally, it has been noted in condensing heat transfer that it is necessary to have pressure gradient information of unusually high accuracy in order to estimate the shear velocity.

Experimental studies of condensing abound, but relatively few are as complete as the investigation of Goodykoontz and Dorsch (*1*). These workers considered the condensation of very high velocity steam in a downward vertical tube and monitored local pressure gradients in addition to local heat flux rates. No model was suggested by them to analyze their heat transfer results.

Although rational design procedures for condensers do exist, those based on conservation equations of mass, momentum, and energy are most generally found for laminar flows with relatively few contributions on the flow of turbulent condensates. Indeed where turbulent condensates have been analyzed, the results are often empirical or semiempirical; for example, see references 3 to 5. Exceptions may be found in the work of Kutateladze (*6*), of Rohsenow (*7*), of Bae et al. (*8*), of Kunz and Yerazunis (*9*), and of Altman et al. (*10*). All of these workers present analyses based on boundary-layer results either as a two- or three-region model of the velocity profile. The present results are an extension of the method of Altman et al. (*10*) in which a three-region approximation is made to the boundary-layer velocity profile according to the well-known Martinelli analogy (*11*). In spite of the basic similarity of approach, apparently minor changes in the formalism produce significant variety in the form of solution obtained. In particular, Bae et al. (*8*) have asserted a specific form of linear shear stress profile to act across the film and this leads to complications in the form of T^+ equivalent to Equation (*3c*) (their Equation *23c*).

Since it is imperative to know the magnitude of the shear velocity v_*, an attempt was made to compute it from existing correlations of two-phase pressure drop for adiabatically derived data. We chose the correlation scheme of Lockhart and Martinelli (*12*) and Martinelli and Nelson (*13*) as our basis. We utilized the description of this scheme formulated by Wallis (*14*). However, recently several workers have realized the significance of the cross-stream condensing component on the pressure drop as compared to two-phase systems in which no mass transfer occurs (*15* to *17*). In a condensing flow the friction pressure gradient is higher than for an adiabatic flow at the same mass rate and quality (and conversely lower for an evaporating film). This effect is in addition to the normally accounted for momentum changes between streams. This fact means that the Martinelli-Lockhart-Nelson model of the friction pressure gradient may be inappropriate in condensing flows without modification. Soliman et al. (*18*) have developed equations for the prediction of the condensing heat transfer coefficient in which the Martinelli-Lockhart-Nelson correlation is explicitly incorporated. The effects of this procedure are masked, however, by subsequently fitting their model with empirically determined constants. In addition, they implicitly define a quantity proportional to a shear velocity in manner similar to Equation (1), but in which the total pressure gradient is used rather than the frictional component.

EXPERIMENT

A test condenser for steam was constructed as shown schematically in Figures 1 and 2. The hydraulic loop which was constructed produced steam at rates from 0 to 150 lb./hr., at pressures from 0.5 to 25 lb./sq.in.abs. Volumetric measure-

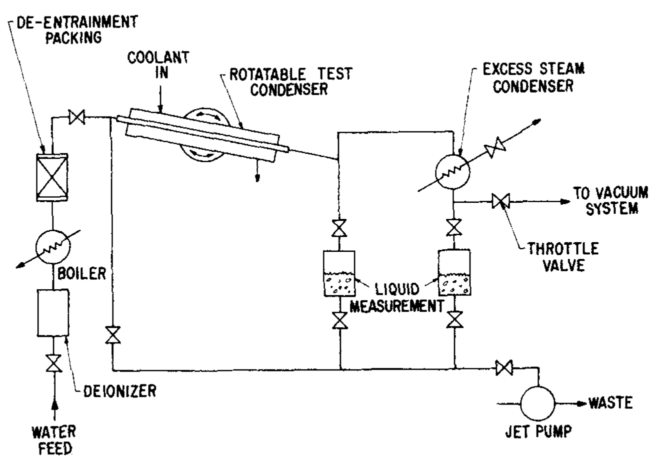

Fig. 1. Simplified condenser loop circuit.

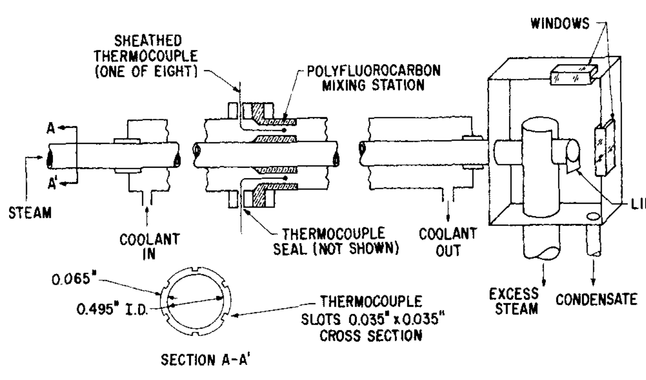

Fig. 2. Schematic diagram of condenser tube.

ment of condensate provided the means of determining the steam flow rates. Any uncondensed steam was condensed in a secondary condenser and its condensate load was measured independently of the primary condensation. Although in normal operation the rig was leak-tight, gaseous noncondensibles could be vented through a steam ejector. Liquid products were pumped to drains by a water-operated jet pump. This condenser rig had two completely novel features, being completely rotatable from the horizontal to vertical positions and equipped with windows at either extremity for visual inspection of flow regimes.

This article will report only on data taken in the annular regime with the condenser tube horizontal. The details of the condenser tube itself are shown in Figure 2. The copper condenser tube of I.D. 0.495 in. was 99½ in. long with an actively cooled section of 86.9 in. Wall temperatures were measured at seven axial locations by sheathed thermocouples soft soldered into appropriately positioned slots in the tube wall. Except at the ends of the condenser, two thermocouples were spaced 180 deg. apart at each axial location. Conduction errors require corrections for estimated inside wall temperatures of ($6.23 \times 10^{-5}\, q''$) °F. [$q''$ in B.t.u./(hr.)(sq. ft.)]. This calculation ignores distortion effects of isothermals in the tube wall due to the presence of the thermocouples. Thermocouple tip positioning errors are approximately ($1.6 \times 10^{-6}\, q''$) °F.

To obtain local condensate rates, it is necessary to measure the coolant temperature rise in the coaxial annular coolant channel. To achieve this in a horizontal tube at several axial locations, and to achieve a measurable ΔT over a range of condensate flows from 0.5 to 150 lb./hr., it is necessary at many operating conditions to account for the radial temperature differences in the coolant arising from natural convective buoyancy. This is achieved by placing polyfluorocarbon mixing stations at each of four axial locations (Figure 2). A cluster of eight sheathed copper-constantan thermocouples is located with one to each of 8 × ¼ in. holes drilled through the mixing stations. Thus the average temperature rise across each of the segments of condenser was determinable and a considerable degree of mixing was achieved between each section.

Sufficient polyurethane foam insulation was provided to render the heat leaks negligible under all conditions reported here.

Within about an inch of the ends of the cooled section of the condenser tube pressure taps were provided for overall Δp measurement. Transparent plastic lines were used to connect an accurate pressure gauge through appropriate valving to the pressure taps. Appropriately located thermocouples were placed in the steam manifolds at inlet and outlet for determination of steam and condensate temperatures.

The data reported here have mass velocities in the range 2×10^3 to 1.1×10^5 lb.$_m$/(hr.)(sq. ft.) with exit qualities of 0.011 to 0.52 and a pressure range of 0.2 to 1.5 atm. At any given flow rate two or three independent sets of data were taken as a check in consistency. A heat balance was drawn between the total steam condensed and the measured average temperature rise in the coolant measured at the mixing stations. From these data the axial heat flux distribution (by graphical interpolation) and the local steam quality were determined. The steam core temperatures were measured at inlet and outlet and, in those cases in which the pressure drop was small (or negligible to measurable accuracy, that is, < 0.05 lb./sq.in.abs.), the mean saturation temperature was used for the calculation of local steam core temperatures. Hydrostatic head corrections were made to the pressure readings by estimating the liquid holdup in the transparent lines. To within experimental accuracy the measured steam temperatures at inlet and outlet agreed with those estimated from the steam tables. At high flow rates it was necessary to apply Bernoulli head corrections between the vapor pressure corresponding to the steam temperatures measured in the large diameter manifold and the pressure measured in the high velocity condenser tube proper. Furthermore, at high flow rates the pressure changes significantly down the tube length and the saturation temperature (assumed equal to the local bulk steam temperature) is then a more difficult quantity to calculate. In all cases the local heat transfer coefficient is computed by dividing the local heat flux by the local difference between the bulk steam temperature and the wall temperature.

ANALYSIS

Pressure Drop

The Martinelli method (*12, 13*) for computing pressure drop has become the standard technique for two-phase flow. There are many improvements on it too numerous to list. These essentially deal with empirical improvements to the so-called two-phase multiplier under various thermodynamic states and flow conditions.

The total pressure gradient in horizontal flow may be expressed as the sum of two terms: one attributable to the effects of viscous shear, and the other to momentum changes between the two phases.

$$-\left(\frac{dp}{dz}\right)_t = -\left(\frac{dp}{dz}\right)_{fr} - \left(\frac{dp}{dz}\right)_m \tag{6}$$

For fully separated, steady, one-dimensional flow of a two-phase fluid of vapor quality x in a constant area duct, it is simple to show that (*14, 19*)

$$\left(\frac{dp}{dz}\right)_m = -G_T^2 \frac{d}{dz}\left\{\frac{(1-x)^2}{(1-\alpha)}\frac{1}{\rho_L} + \frac{x^2}{\alpha}\frac{1}{\rho_v}\right\} \tag{7}$$

where α, the void fraction, can be obtained from, for example, Equation (8) (*20*).

$$\alpha = \frac{1}{1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_v}{\rho_L}\right)^{2/3}} \tag{8}$$

This equation has been challenged by Andeen and Griffith (*19*) but has the distinction of being derived by plausible arguments especially for separated flow as under investigation here.

The frictional component can be obtained from the separate cylinders model of two-phase pressure drop due to Wallis and co-workers (*14*). This is a semiempirical realization of the original Martinelli-Lockhart-Nelson correlation (*12, 13*). The two-phase frictional pressure gradient is expressed in terms of the vapor-phase pressure gradient measured as if the vapor filled the pipe at the local vapor rate ($-(dp/dz)_g$) times an empirical multiplier (ϕ^2_g).

$$\left(\frac{dp}{dz}\right)_{fr} = \phi_g^2 \left(\frac{dp}{dz}\right)_g \tag{9}$$

where

$$\phi_g^2 = (1 + \underline{X}^{2/n})^n \tag{10}$$

and

$$\underline{X}^2 = \left(\frac{dp}{dz}\right)_f \Big/ \left(\frac{dp}{dz}\right)_g \tag{11}$$

The $\underline{X}^2$ multiplier depends on the ratio of single-phase pressure gradients and is readily calculated by the standard methods. In Wallis' treatment the exponent n was found to depend on the flow regime of the two phases, that is, with both phases turbulent $n = 4$. Under these conditions, when the necessary relationships were substituted into Equation (6) rather large deviations between measured and predicted pressure profiles were noted when using the experimental data of Goodykoontz and Dorsch (*1*); for example, see Figure 3. It was decided to adjust empirically the predictions of the separate cylinders model of Wallis (*14*) to account for this increased frictional pressure drop. Such a device is not of course a general technique, but over the range of conditions experimentally covered, an adequate fit was found by putting $n = 5.13$. Figure 3 shows the subsequent improvement in predicting the pressure profile of a set of data taken from reference *1*. The identical correction was then applied to the data taken in the course of the present experiments and was found to be applicable to a reasonably high degree of accuracy; see Figure 4. A more critical test is shown in Figure 5 which is for a set of data at high pressure drop. The local saturation temperature, and hence condensing temperature difference, has been derived from the local computed pressure profile and also from a linear plot of T_s from the measured inlet to outlet stations. The increase in consistency of h versus axial length is indicative of the increased accuracy with the computed pressure profile.

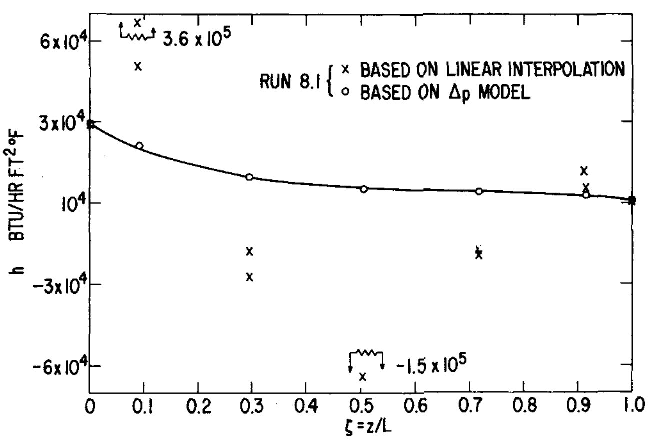

Fig. 5. Effect of assumed pressure gradient on heat transfer coefficients.

Heat Transfer Coefficient

The local heat transfer coefficients, nondimensionalized to Nusselt numbers, hD/k_L, were compared to the empirical formulae of Akers, Deans, and Crosser (*5*) [see Equations (12)] and of Ananiev, Boyko, and Kruzhilin (*3, 4*) [Equations (13)], both of which are backed up by considerable data.

$$N_{Nu} = 0.0265\ N_1^{0.8}\ N_{Pr}^{1/3} \qquad \text{for } N_1 > 50{,}000 \quad (12a)$$

$$N_{Nu} = 5.03\ (N_1\ N_{Pr})^{1/3} \qquad \text{for } N_1 < 50{,}000 \quad (12b)$$

where N_1 is a Reynolds number given by

$$N_1 = \frac{G_T D}{\mu_L} \{1 - x + x(\rho_L/\rho_v)^{1/2}\} \quad (12c)$$

$$N_{Nu} = 0.021\ N_2^{0.8}\ N_{Pr}^{0.43} \quad (13a)$$

where a Reynolds number is defined by the expression

$$N_2 = \frac{G_T D}{\mu_L} \left\{ 1 + \frac{\rho_L - \rho_v}{\rho_v} x \right\}^{0.625} \quad (13b)$$

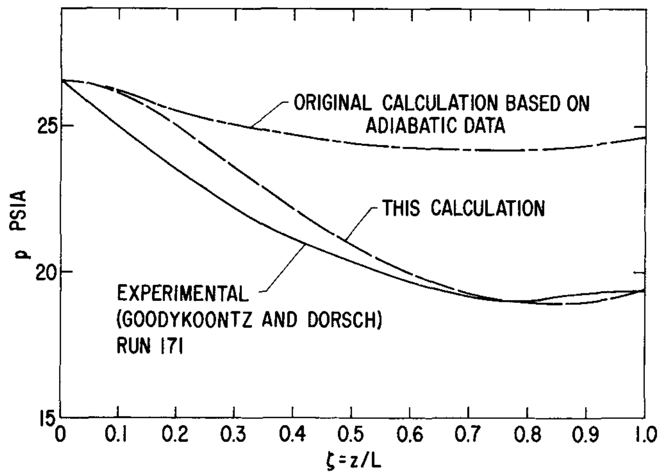

Fig. 3. Condensing pressure drop calculations.

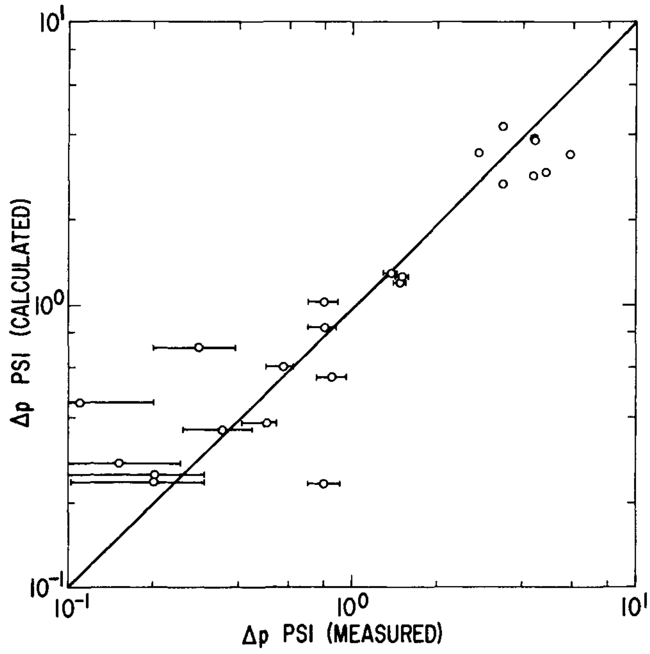

Fig. 4. Comparison of predicted to measured pressure drop in condensation.

The results are shown, respectively, in Figures 6 and 7. These data include a spectrum of observed flow regimes at exit: annular, mist, and stratified. Neither equation adequately predicts local Nusselt numbers. It is for this reason that a more rational analysis to the computation of the heat transfer rate is required.

The starting point for this analysis is that of reference *10*, which in turn is based on the well-known Martinelli analogy for turbulent flow heat transfer. These results may be generalized as follows:

$$N_{Nu} = \frac{\rho_L\, D\, c\, v_*}{k_L\, T^+(\delta^+)} \quad (14a)$$

where

$$T^+(\delta^+) = \delta^+ N_{Pr} \quad \text{for} \quad \delta^+ \leq 5 \quad (14b)$$

and

$$T^+(\delta^+) = 5[N_{Pr} + \ln\{1 + N_{Pr}(\delta^+/5 - 1)\}] \quad (14c)$$

$$\text{for} \quad 30 \geq \delta^+ > 5$$

and

$$T^+(\delta^+) = 5\ \{N_{Pr} + \ln(1 + 5N_{Pr}) + 0.495 \ln(\delta^+/30)\} \quad \text{where} \quad \delta^+ > 30 \quad (14d)$$

The most important result in reference *10* is an expression for the film thickness δ. It is obtained by integrating the 1/7th power law velocity distribution from the wall to thickness δ. A nondimensional generalization of this quantity is given in reference *21*. Similar results are available elsewhere (for example, see reference *14*). It is shown (*21*) that

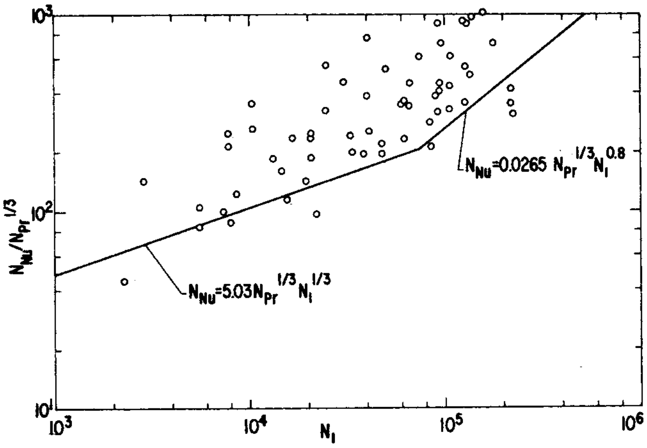

Fig. 6. Local condensation Nusselt numbers compared to empirical correlation of Akers, Deans, and Crosser.

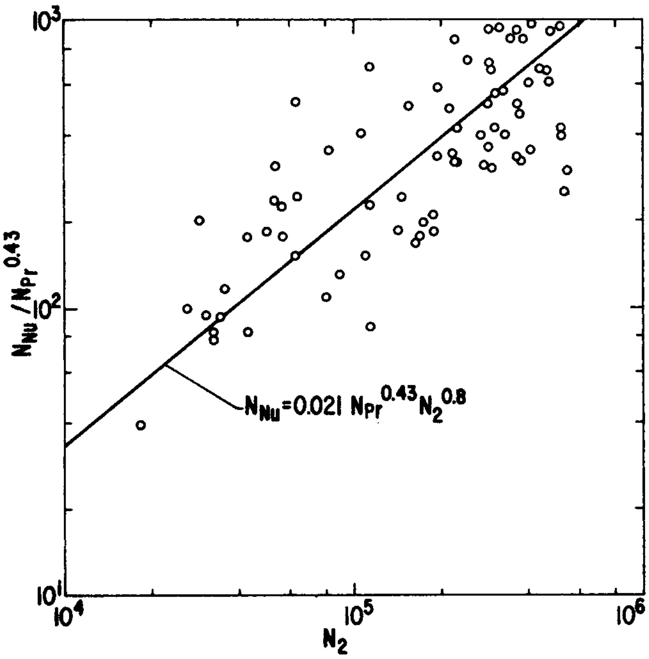

Fig. 7. Local condensation Nusselt numbers compared to empirical correlation of Ananiev, Boyko, and Kruzhilin.

$$\delta^+ = (N_{Re}/2)^{1/2} \text{ for } N_{Re} < \sim 1{,}000 \quad (15a)$$

and

$$\delta^+ = 0.0504\, N_{Re}^{7/8} \text{ for } N_{Re} > \sim 1{,}000 \quad (15b)$$

These equations are based, respectively, on the integration of the laminar sublayer profile ($v^+ = y^+$) and Prandtl's 1/7th law ($v^+ = 8.74\, y^{+1/7}$). The relevant Reynolds number is given by

$$N_{Re} = 4\int_0^{\delta^+} v^+\, dy^+ = (1-x)\frac{G_T D}{\mu_L} \quad (16)$$

For the shear-dominated annular flow reported here, the shear velocity is simply defined as previously given:

$$v_* = \sqrt{\frac{-D}{4\rho_L}\left(\frac{dp}{dz}\right)_{fr}} \quad (1)$$

The frictional component of the pressure gradient is then obtained from the modified Wallis (*14*) treatment of the two-phase pressure gradient as previously described. In Figure 8 Equations (14) and (15) are combined and compared to experimental data taken in the annular flow regime in this series of experiments. The range of δ^+ covered in the data reported is given in Table 1. Thus, as the condensate progressed along the tube length, all the flow regimes described by the limits of Equation (14) were applicable.

A further comparison is given from the data of Altman et al. (*10*) for the condensation of refrigerant 22. These data are taken at a saturation temperature of 100°F. (reduced pressure 0.28) and cover a mass velocity range of 2.2 to 6.3 × 10^5 lb.$_m$/(hr.)(sq.ft.) with local mean vapor qualities varying between 0.28 and 0.92. At these conditions the flow was believed to be in the annular regime.

DISCUSSION

It is clear from Figures 6, 7, and 8 that an improvement in prediction of the local condensing heat transfer rates is obtained by application of the Martinelli analogy in the manner prescribed. This point is emphasized in Figure 9. Data from a single run are shown so that the substantial variation of h with quality (or equivalently condenser length) is obvious. Given for comparison purposes are the results derived from Equations (12) and (13) of Akers et al. (*5*) and of Ananiev et al. (*3, 4*), respectively. These equations fail to predict the highly significant variation of the local heat transfer coefficient with length which is characteristic of condensation with high velocity inlet conditions. Finally, note that the magnitude of the variation of h with length demands the implementation of local

TABLE 1. FILM THICKNESS AT CONDENSER TUBE EXIT

Run No.	δ^+ at exit
16.1	33.9
19.3	79.7
20.4	41.6
25.2	120
30.1	41.5

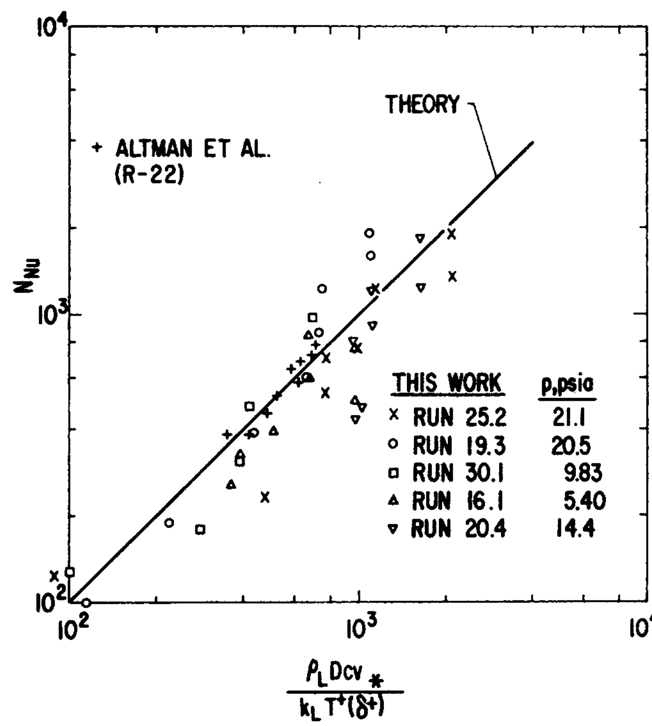

Fig. 8. Test of annular theory for condensation.

design methods rather than the use of average coefficients if high accuracy is required.

An approximation introduced into the analysis is the use of Equation (15) as a representation of the nondimensional thickness of the thin film. There are several oversimplifications implied thereby which are discussed in reference *21*. In the idealized sense annular flow is a uniform steady film of condensate flowing on the internal tube walls irrespective of the direction of the gravitational vector. In real flows chugging, rippling, traveling waves, and entrainment occur. These were all ignored in the derivation of Equation (15) whose justification is *a posteriori*. Satisfactory agreement was noted in reference *21* for Equation (15) over a considerable spectrum of experimental data.

A very important point has been made in Equation (1). It is there suggested that the correct value of the shear velocity follows from the frictional component of the pressure gradient. This view is in agreement with that of Altman et al. (*10*) but is contrary to that of Soliman et al. (*18*) whose correlation of the condensing heat transfer coefficient is based upon $-(dp/dz)_t$. Their view was challenged in the discussion section of their paper by Rohsenow whose procedure, in another report (*8*), was very similar in principle to the one followed here. To emphasize the correctness of the viewpoint put forward here, Figure 10 was drawn. These data are those of Goodykoontz and Dorsch (*1*) (run 228) in which a static pressure rise was noted over the condensing length. This is due to a low frictional pressure drop and a high momentum pressure recovery. The Soliman et al. (*18*) recommendation leads to $v_* = (-D(dp/dz)_t/4\rho_L)^{1/2}$. In this case $(dp/dz)_t > 0$ and v_* would be meaningless. To generate the theoretical line in Figure 10, Equation (1) was used in conjunction with Equations (14) and (15). In this particular calculation the void fraction correlation for use in Equation (7) was that of Martinelli (*12, 13*).

It is strongly believed that the success in fitting data claimed by Soliman et al. (*18*) is due to the fact that the momentum component of the pressure gradient was rather small compared to the frictional component in the cases tested by them.

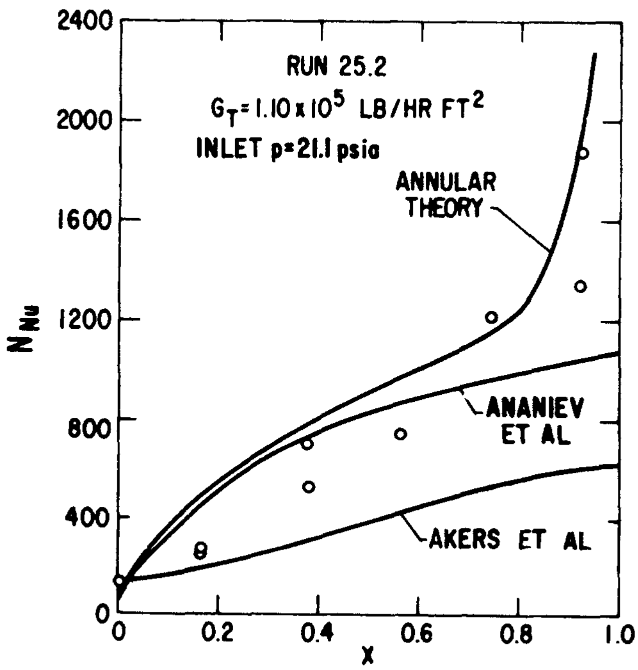

Fig. 9. Axial variation of local Nusselt numbers.

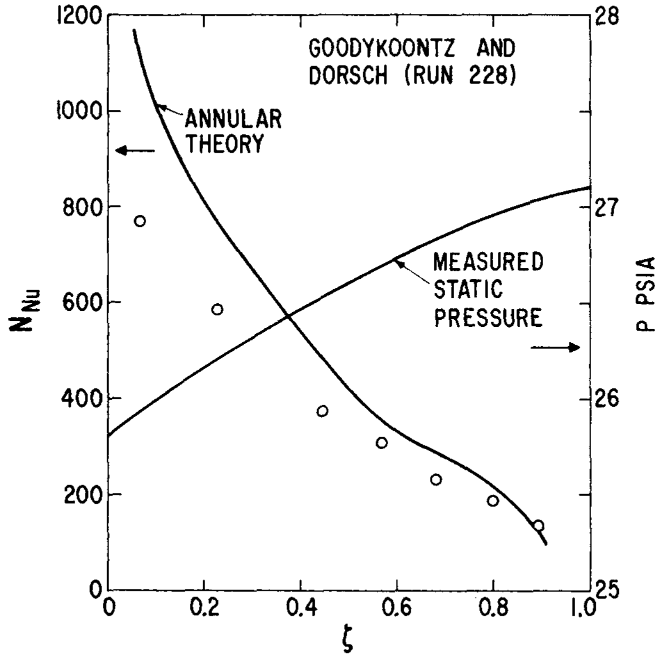

Fig. 10. Prediction of Nusselt number for case in which static pressure rises.

Finally it has been noted that the condensing pressure drop was considerably in excess of the Martinelli two-phase adiabatic correlation. It is perhaps instructive to point out the reason why such large effects (that is, ~ −300% in Figure 3) have been neglected despite a major effort in the study of two-phase pressure drop. Much of this effort has been directed toward boiling and, in that case, the momentum pressure change has the same sign as the friction pressure drop. As a concrete example, consider the following assumed cases:

Boiling	Condensing
$\Delta p_m = 3$ lb./sq. in.	$\Delta p_m = -3$ lb./sq. in.
$\Delta p_{fr} = 5 \pm 2$ lb./sq. in.	$\Delta p_{fr} = 5 \pm 2$ lb./sq. in.
$\Delta p_t = 8 \pm 2$ lb./sq. in. (±25%)	$\Delta p_t = 2 \pm 2$ lb./sq. in. (±100%)

The error limits in Δp_{fr} are realistic for the computation based on the Martinelli method (*22*). The propagation of the error is most important when the momentum pressure gain is numerically of similar magnitude to the frictional pressure loss.

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NOTATION

c = specific heat of liquid
D = internal diameter of tube
G_T = mass velocity, that is, total mass rate of flow per unit area
h = heat transfer coefficient, $q''/(T_S - T_W)$
k_L = thermal conductivity of liquid
L = length of condenser

n = empirical exponent dependent on flow regimes in the two phases
N_{Nu} = hD/k_L, Nusselt number
N_{Pr} = $(\mu c/k)_L$, Prandtl number for liquid
N_{Re} = $(1 - x)G_T D/\mu_L$
N_1 = $G_T D/\mu_L \cdot \{1 - x + x(\rho_L/\rho_v)^{1/2}\}$
N_2 = $G_T D/\mu_L \cdot \{1 + (\rho_L - \rho_v)x/\rho_v\}^{5/8}$
p = pressure
q'' = heat flux rate based on inside tube area
T = temperature
$T^+(\delta^+)$ = dimensionless temperature
T_S = saturation temperature of vapor
T_W = inside wall temperature
v = local velocity in liquid film
v_* = shear velocity
v^+ = v/v_*
x = vapor quality, that is, local mass fraction of vapor to total flow
z = axial coordinate from tube inlet

Greek Letters

α = void fraction
δ = film thickness
δ^+ = dimensionless film thickness $\delta v_*/\nu_L$
ζ = dimensionless axial location z/L
μ_L = dynamic viscosity of liquid
ν_L = kinematic viscosity of liquid
ρ_L = density of liquid
ϕ = two-phase multiplier, ratio of two-phase frictional pressure gradient to single phase flowing only
$\underline{X}$ = two-phase multiplier, ratio of local frictional pressure gradients of the two phases

Subscripts

f = liquid phase flowing at mass velocity $(1 - x)G_T$
fr = frictional
g = vapor phase flowing at mass velocity $x\, G_T$
m = momentum
t = total static

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Countercurrent Equilibrium Stage Separation with Reaction

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SCOPE

A number of reactions of commercial interest are limited in their attainable extent of conversion by unfavorable reaction equilibrium. The most common schemes for overcoming this limitation are to feed one of the reactants, generally the cheapest one, in great excess or to separate and recycle the unconverted reactants in the reactor effluent or a combine of both methods. In some cases it is possible to remove continuously one or more of the products from the reaction zone and thereby drive the reaction

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